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Institute for Experiential Robotics at Northeastern

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Online Refinement of Uncertainty Sets for Robust MPC of Quadrupedal Robots Using Convex Cone Programming

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Problem Statement

- Given a pre-defined contact sequence, generate optimal ground reaction forces for the stance feet.
- Track a desired position and velocity profile for the Center-of-Mass.
- Ensure stable walking motion under inertial, contact location, impact model and frictional uncertainties with imperfect bounds.

Algorithm

For bounded uncertainty in inertia , contact point & impact model, linearized Single Rigid Body Dynamics can be expressed as:

$$\begin{aligned} x_{i+1} &= A_i x_i + (B_i + \Delta B_i)(u_i + \Delta u_i) \\ &= A_i x_i + B_i u_i + E\theta_i^a \end{aligned}$$

$E\theta_i^a$: Time-varying cumulative effects of modeling uncertainties

Step 1 – Online Dynamics Uncertainty Refinement:

Refine domain of $E\theta_i^a$, $\Omega = \{E\theta_i : H(E\theta_i) \leq h\}$ based on history of past robot motion, $\{A_0, \ldots, A_{-p}, B_0, \ldots, B_{-p}, x_0, \ldots, x_{-p}, u_0, \ldots, u_{-p}\}$ [3]

Define control law and uncertain friction cone constraints parametrized by ρ [1]

$$u_i \in \mathbb{R}^{12} = \underbrace{K_i E\theta_i}_{\text{feedback term}} + \underbrace{v_i}_{\text{feedforward term}}, \quad \begin{aligned} &(C_{\text{nom}} + C_{\text{dist}} z)u_i = (C_{\text{nom}} + C_{\text{dist}} z)(K_i E\theta_i + v_i) \leq 0 \\ &z \in \mathcal{Z} \triangleq \{z \in \mathbb{R} \mid -\rho \leq z \leq \rho\} \end{aligned}$$

Step 2 – Solve QP-MPC with nominal dynamics and tightened constraints:

$$\min_{x[.],v[.],\lambda^+[.]} \sum_{i=0}^{N-1} \left(||x_i - x_{i,\text{desired}}||_Q^2 + ||v_i||_R^2 \right) \quad \text{MPC Cost-Function}$$

$$\gamma_i = \max_{\theta_i \in \Omega} \; C_{\text{nom}} K_i E\theta_i$$

$$\theta_i^* = \arg\max_{\theta_i \in \Omega} \; C_{\text{nom}} K_i E\theta_i$$

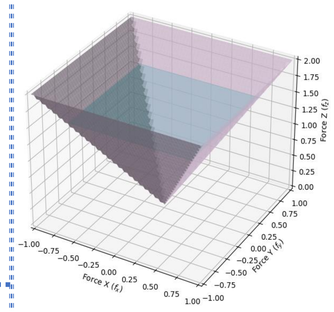
$$\boxed{\begin{aligned} &\gamma_i + C_{\text{nom}} v_i + \lambda_i^\top \alpha\rho \leq 0 \\ &\lambda_i^\top \beta - C_{\text{dist}}(K_i E\theta_i^* + v_i) = 0 \end{aligned}} \longrightarrow \text{Tightened MPC Constraints}$$

$\longrightarrow$ Linear Program 1

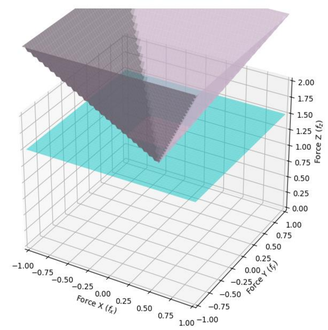
Step 3– Online Friction Uncertainty Refinement: Get smallest ρ satisfying MPC solution

$$\rho^* = \min_{\rho} \rho \text{ , s.t. } \quad \gamma_i + C_{\text{nom}} v_i^* + {\lambda_i^*}^\top \alpha\rho \leq 0, \; \forall i = 0 \ldots N-1 \longrightarrow \text{Linear Program 2}$$

Adjusted Friction Cone

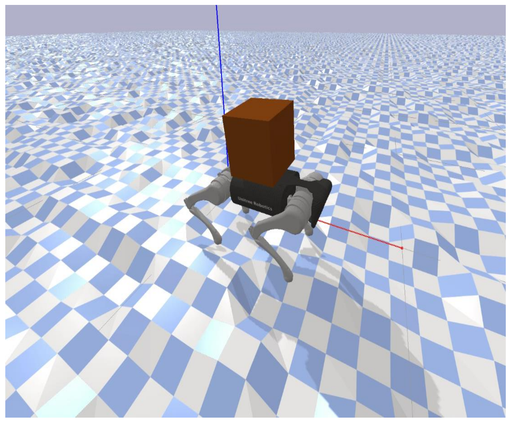


Nominal Friction Cone

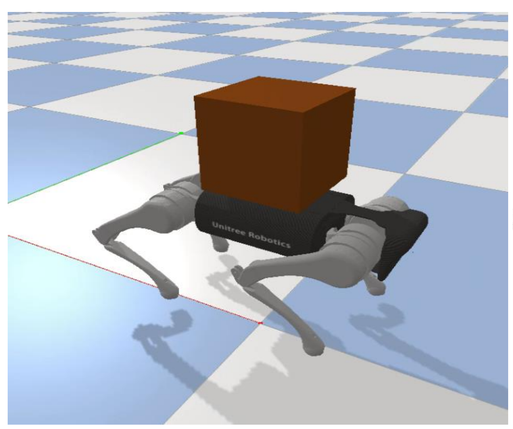


Tightened Friction Cone

Constraint Tightened MPC (without adaptation)

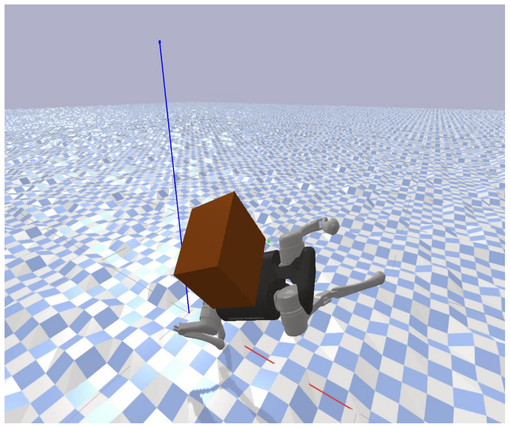


Uneven Terrain with 5 kg payload

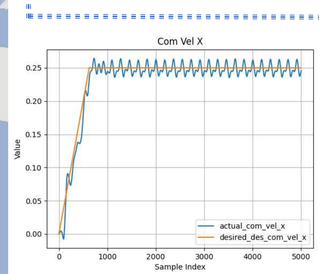


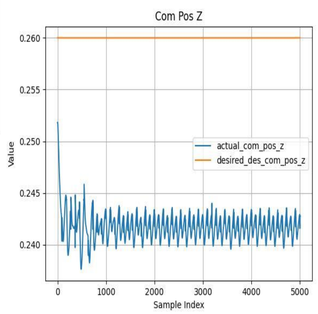
Flat Terrain with 5 kg payload

Preliminary Results



Nominal MPC [2]





Accurate height and velocity tracking

[1] Robust Convex Model Predictive Control for Quadruped Locomotion Under Uncertainties, Xu et al
[2] Dynamic Locomotion in the MIT Cheetah 3 Through Convex Model-Predictive Control , Di Carlo et al
[3] Adaptive MPC under Time Varying Uncertainty: Robust and Stochastic, Bujarbaruah et al